THE BULLWHIP EFFECT IN INDUSTRY – A MONTE-CARLO SIMULATION STUDY

Hans - J. Lenz, Global Business Intelligence Server Research Group, Institute of Production, Information Systems and Operations Research, Freie Universität Berlin, D
hans-j.lenz@fu-berlin.de

Emilia Paustian, Department of Economics, Freie Universität Berlin, D
emilia.paustian@gmx.de

Abstract

We analyze artificial business cycles of main business indicators caused by SCM software, explain the mechanism behind this phenomenon, and suggest some hints how to prevent such bad behaviour. The bullwhip effect is not an academic artefact, but happened for instance to CISCO when the company firstly introduced SCM software from the vendor SAP to manage its supply chain. Remember that a supply chain is an alliance of firms acting mutually as consignee and supplier. Such a chain may include farmers, co-operatives, a manufacturers, retailers and consumers. Modelling such phenomena leads to a multi-stage, stochastic, interdependent multi-equation system which is not analytically tractable except for unrealistic assumptions about the final demand rate. The mathematical intractability is mainly caused by the positive lead time of order and production. This is a generic characteristic of the dominating differential or difference equation and describes the dynamics of the inventory on each stage and time. Note that on each stage the true demand rate for a given period is not at hand, but only its expectations or beliefs are available. Therefore we build a sophisticated model of a supply chain and solve it by MC simulation to allow for any reasonable kind of seasonal and non seasonal time series patterns of the end user's demand rate, prediction formula, inventory strategy, and information level.

Keywords: Supply chain management, Monte-Carlo Simulation, multi-stage planning system.

1 INTRODUCTION

First of all, we introduce the mathematical nature of a supply chain. We restrict ourselves to a four stage system. It consists of a consumer, retailer, dealer and producer / manufacturer. We are not interested in the management of such a chain, i.e. neither in establishing a chain using IT, running nor managing it. Quite opposite, we aim for asking the question why and how it could happen that the main indicators of firms being members of a supply chain can suffer from cyclic behaviour (Bullwhip Effect) when using standard SCM software even if the end user orders the same quantity each period and there happens only a single shock. This effect may be quite surprising for managers to belief that a single spike in a constant demand rate can cause a cyclic behaviour of cost or inventory. Such a bad experience made by Cisco in 2001 was reported by (Kaihla 2002). Furthermore, is it possible that the variance of the stationary, even non-seasonal demand rate at the producer's site is eight times larger than the variance of the end user's demand rate, cf. scenario 2 below?

It should be stressed that the trivial case of a constant demand rate with a single shock at the consumer's site can be analytically treated, cf. (Warburton 2004). To the best of our knowledge an analysis of a supply chain assuming a more realistic demand rate is prohibitive. This is due to the dominating differential equation which has a non-zero delay parameter and prevents a full mathematical treatment of the problem. In the following we assume a backlog case for an overshooting demand rate with respect to the stock at hand.A proxy was given by (Essebbabi and Lenz 2006) who substitute the stationary random demand process D_t, t= 1,2,…, of type AR1 (autoregressive process of first order) by its unconditional expectation. This proxy enables them to study the influence of two important parameters which cause the bullwhip effect, variance $V(D_t)$ and first-order auto-correlation $\rho_D(1)$ of the end user's demand rate D_t.

In our study we characterize the demand process by an ARIMA process, we analyze the supply chain system in detail on each stage, and describe the experimental set-up for running interesting scenarios based on MC simulation which hopefully will give the practioner hints what might happen to main business indicators when SCM software is in use. The Bullwhip effect characterizes how much the production rate X of a manufacturer is deviating according to the final demand rate D_t. It can be measured by the ratio $Var(X_t)/Var(D_t)$, cf. (Chen et al. 2000). (Keller 2004) proved that the demand rate on each supply chain stage may not converge to a new level considering a horizon of forty periods but is characterized by a steadily increasing variance $Var(D_t)$. An improved definition of the Bullwhip effect considers the dependency of the varying order rates on the inventory level I_t, see (Disney and Towill 2003). This view leads to the ratio $Var(I_t)/Var(D_t)$. (Lee et al. 1997) and (Duc et al. 2008) make the conjecture that the selection of a prediction formula mainly influences the bullwhip effect because the predictor directly influences the inventory system.

In the following we recollect the main items of SCM, i.e. demand process and supply chain system. Consequently, we explain our set-up for MC simulation. The main results of our investigations are bundled together in five scenarios. All of them look at different sides of the problem and contribute to unfold the risks inherent in a bad supply management.

2 DEMAND PROCESS

We consider a temporal sequence of demand rates as a process with an autoregressive (AR) component of order p and a moving average (MA) component of order q. This mixing is called autoregressive-moving average process, abbreviated as ARMA(p,q). It can be written in structural form as (p=q=1)

(1)
$$D_t = \delta + \alpha D_{t-1} + u_t - \beta u_{t-1}$$
$$\text{with } E(u_t) = 0 \text{ and } E(u_t, u_s) = \begin{cases} \sigma^2 & \text{for } t = s \\ 0 & \text{for } t \neq s \end{cases}$$

or in reduced form:

(2)
$$D_t = \frac{\delta}{1-\alpha} + \frac{1-\beta L}{1-\alpha L} u_t .$$

Identification of the parameters is guaranteed iff stationarity and invertibility conditions are fulfilled: $\alpha \neq \beta$, $\frac{\delta}{1-\alpha} < \infty$, and $|\alpha| < 1$. Note that using an AR_1MA_1 and appropriately fixing α, β we can emulate a stochastic (pseudo) seasonal pattern of the demand rate D_t.

As a predictor of the demand rate $D_{t+\tau}$ in period t for a lead time (prediction horizon) τ-steps ahead we use $\hat{D}_t(\tau)$, i. e. the conditional expectation of $D_{t+\tau}$ in t.

(3)
$$\hat{D}_t(\tau) = E(D_{t+\tau} | D_t, D_{t-1}, D_{t-2},...) = E_t(D_{t+\tau}) \quad \text{for } \tau = 1,2,... .$$

We get as one-step ahead predictor:

(4) $$\hat{D}_t(1) = \delta + \alpha D_t - \beta u_t$$

and as a two-step ahead predictor:

(5) $$\hat{D}_t(2) = \delta + \alpha \hat{D}_t(1).$$

The predictor can be evaluated for a general lead time τ≥1:

(6 $$\hat{D}_t(\tau) = \delta + \alpha D_{t+\tau-1} + u_{t+\tau} - \beta u_{t+\tau-1} = \frac{1-\alpha^\tau}{1-\alpha}\delta + \alpha^\tau D_t - \alpha^{\tau-1}\beta u_t.$$

From $\mu = \frac{\delta}{1-\alpha}$ follows:

(7) $$\hat{D}_t(\tau) = \mu(1-\alpha^\tau) + \alpha^\tau D_t - \alpha^{\tau-1}\beta u_t.$$

Identifiable conditions of ARMA processes of orders p, q generally are fulfilled iff $\alpha_1, \alpha_2, ..., \alpha_p \neq 0$ and $\beta_1, \beta_2, ..., \beta_q \neq 0$. Moreover, the polynomials in the backward shift operator L, $\alpha_p(L)$ and $\beta_q(L)$, are not allowed having common factors and the lag polynomials must have roots outside the unit circle.

3 SUPPLY CHAIN SYSTEM

We consider a four stage chain: Consumer, retailer, gross dealer and a producer, cf. Figure1. At the retailer level the inventory or stock at the beginning of period *t* is increased by incoming orders, and will decrease when the supply rate was effective, i.e. the customers have made their orders. According to this information a decision about ordering can be made at the end of the current period (*t-1*, *t*]. This order is transmitted to the (preceding) firm on the next higher level of the chain. In the following section the levels and the co-located decisions are represented by appropriate equations.

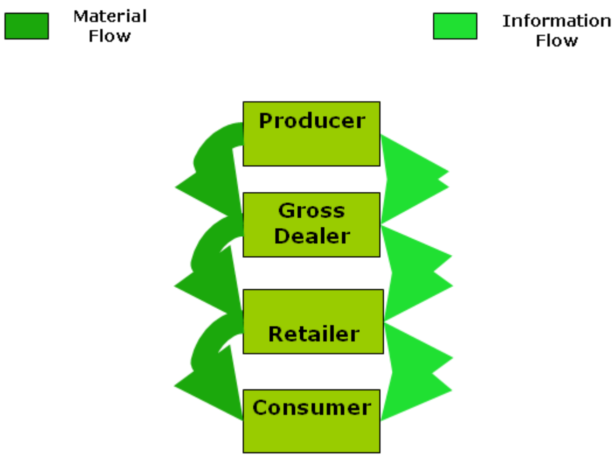

Figure 1. Supply Chain with four levels

The order O_t^E of the retailer at time t is the usual linear combination of the predicted demand rate $\hat{D}_t$ of the end user, the work-in-process WIP_t^E and inventory I_t^E:

(8) $$O_t^E = \max\{0, \hat{D}_t - WIP_t^E - I_t^E\}$$

The gross dealer G needs ρ^G periods to supply and ship the quantity ordered by the retailer. Therefore the lead time for the predictor τ is determined by the supply time (delay):

(9) $$\hat{D}_t = \sum_{\tau=1}^{\rho^G} \hat{D}_t(\tau) = \sum_{\tau=1}^{\rho^G} \left(\mu(1-\alpha^\tau) + \alpha^\tau D_t - \alpha^{\tau-1}\beta u_t\right)$$

$$= \mu \sum_{\tau=1}^{\rho^G} (1-\alpha^\tau) + D_t \sum_{\tau=1}^{\rho^G} \alpha^\tau - \beta u_t \sum_{\tau=1}^{\rho^G} \alpha^{\tau-1}$$

$$= \mu\left(\rho^G - \frac{\alpha(1-\alpha^{\rho^G})}{1-\alpha}\right) + \frac{\alpha(1-\alpha^{\rho^G})}{1-\alpha} D_t - \frac{(1-\alpha^{\rho^G})}{1-\alpha} \beta u_t$$

$$= \mu\rho^G - \mu\alpha\varpi^E + \varpi^E(\alpha D_t - \beta u_t),$$

where $\varpi^E = \frac{(1-\alpha^{\rho^G})}{1-\alpha}$.

The work-in-process (running or open orders) is subtracted from the predicted demand rate $\hat{D}_t$ given in (9). The running orders are those orders which are not still satisfied and which will be shipped in the future:

(10) $$WIP_t^E = \begin{cases} 0 & \text{für } \rho^G = 1 \\ \sum_{i=1}^{\rho^G-1} O_{t-i}^E & \text{sonst} \end{cases} \qquad \text{für } \rho^G = 1,2,\ldots.$$

Furthermore, the predicted demand rate must be reduced by the stock at hand, I_t^E:

(11) $$I_t^E = I_{t-1}^E + O_{t-\rho^G}^E - D_t \, .$$

As mentioned above we assume a backlog case. This implies that the supply of the gross dealer is equal to the order by the retailer issued ρ^G periods ahead, i.e. $O_{t-\rho^G}^E$. Simply speaking the demand of the end user, D_t, is satisfied.

It is not reasonable to reduce the bullwhip effect to statistical indicators only. From the management point of view it makes more sense to consider economical indicators of the bullwhip effect, i.e. cost. They are given by

(12) $$K_t^E = h^E L_t^E + b^E O_{t-\rho^G}^E - f^E F_t^E,$$

with $L_t^E = \max\{0, I_t^E\}$

and $F_t^E = \min\{0, I_t^E\}$.

Evidently, the cost of the dealer in t are added up by inventory cost h^E per unit of the dealer multiplied by his (positive) stock (inventory) I_t^E, and the varying cost of ordering b^E per unit multiplied by the orders arriving in t and, finally, the penalty cost f^E per unit for undershooting, F_t^E.

In a similar way we can use the same order equation for the gross dealer. The main difference is only that here the „driving force“ is the ordered quantity of the retailer, and not of the consumer:

(13) $$O_t^G = \max\{0, \hat{O}_t^E - WIP_t^G - I_t^G\}.$$

It is obvious and plausible that the gross dealer does not know the true demand rate of the end user. It is unknown to him due to lack of information. Instead, he has to use information only derived from the incoming orders at his site. Consequently the gross dealer has to make predictions over the supply time of the producer, i.e. his supplier.

(14) $$\begin{aligned}\hat{O}_t^E &= \sum_{\tau=1}^{\rho^P} \hat{O}_t^E(\tau) = \sum_{\tau=1}^{\rho^P} \left[\mu(1-\alpha^\tau) + \alpha^\tau O_t^E - \alpha^{\tau-1}\beta u_t\right] \\ &= \mu\sum_{\tau=1}^{\rho^P}(1-\alpha^\tau) + O_t^E \sum_{\tau=1}^{\rho^P}\alpha^\tau - \beta u_t \sum_{\tau=1}^{\rho^P}\alpha^{\tau-1} \\ &= \mu\left(\rho^P - \frac{\alpha(1-\alpha^{\rho^P})}{1-\alpha}\right) + \frac{\alpha(1-\alpha^{\rho^P})}{1-\alpha} O_t^E - \frac{(1-\alpha^{\rho^P})}{1-\alpha}\beta u_t\end{aligned}$$

We can summarize (14) as:

(15) $$\hat{O}_t^E = \mu\rho^P - \mu\alpha\varpi^G + \varpi^G(\alpha O_t^E - \beta u_t),$$

where $\varpi^G = \frac{(1-\alpha^{\rho^P})}{1-\alpha}$.

The work-in-process and the inventory are not affected:

(16) $$WIP_t^G = \begin{cases} 0 & \text{für } \rho^P = 1 \\ \sum_{i=1}^{\rho^P - 1} O_{t-i}^G & \text{sonst} \end{cases} \quad \text{für } \rho^P = 1,2,\ldots$$

(17) $$I_t^G = I_{t-1}^G + O_{t-\rho^P}^G - O_t^E.$$

Even the cost function is kept the same:

(18) $$K_t^G = h^G L_t^G + b^G O_{t-\rho^P}^G - f^G F_t^G,$$

with $L_t^G = max\{0, I_t^G\}$

and $F_t^G = min\{0, I_t^G\}$.

The producer is expected to make his decisions at the end of each period as the rest of the chain members do, too. As the manufacturer represents the final stage of a chain, he cannot order from a third party but has to control his production rate.

(19) $$X_t = max\{0, \hat{O}_t^G - WIP_t^P - I_t^P\}.$$

The producer's predictor for the gross dealer's order rate is:

(20) $$\hat{O}_t^G = \mu\lambda - \mu\alpha\varpi^P + \varpi^P(\alpha O_t^G - \beta u_t),$$

where $\varpi^P = \frac{(1-\alpha^{\lambda})}{1-\alpha}$.

The parameter λ is the epoch, which the producer needs to produce the ordered quantity X_t. The subset which is currently manufactured is represented by the work-in-process WIP_t^P:

work-in-process: $$WIP_t^P = \begin{cases} 0 & für\ \lambda = 1 \\ \sum_{i=1}^{\lambda-1} X_{t-i} & sonst \end{cases} \qquad \text{für } \lambda = 1,2,\ldots$$

The inventory in t is analogy defined as above:

(21) $$I_t^P = I_{t-1}^P + X_{t-\lambda} - O_t^G.$$

The cost equation is given by

(22) $$K_t^P = h^P L_t^P + b^P X_{t-\lambda} - f^P F_t^P$$
with $L_t^P = \max\{0, I_t^P\}$
and $F_t^P = \min\{0, I_t^P\}$.

We close with a summary of the complex system of equations in Figure 2. which are related to the four levels of the supply chain.

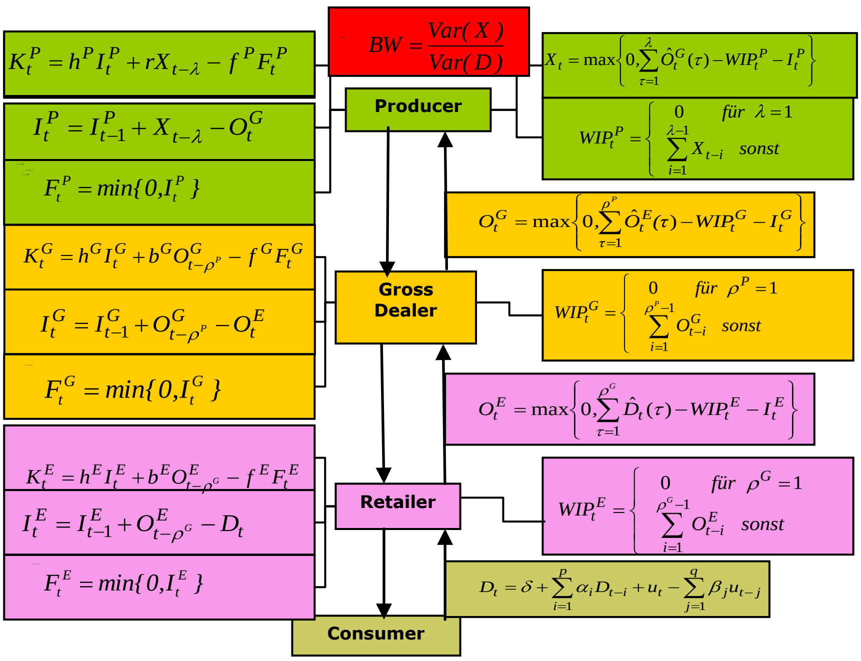

Figure 2. A four level supply chain with its main functional relationships

4 MC SIMULATION

Given the above model of a supply chain we are now able to study the chain effects on interesting business indicators. The driving forces or influencing factors of the chain system are: The type and the parameters of the demand process D_t, the predictor used for forecasting demand, the delivering delay (lead time) ρ and the order or production time λ. We combine these parameters with five scenarios of interest.

4.1 Experimental Set-up

The factor levels of the demand process type are: constant rate, stationary ARMA process with / without pseudo-seasonal behaviour, and a non-stationary AR process. A further influencing factor is the predictor. For the sake of simplicity of presentation we stick to quadratic-minimum (MMSE), single exponential smoothing or moving average predictors, cf. (9). The response of interest is mainly the Bullwhip effect measured as a ratio of variances of demand rates, but further business indicators like the order quantity, the inventory level or the total cost of a supply chain management, TCSC, are of interest, too.

4.2 Scenarios

Altogether we analyze five scenarios in detail which unscramble different facets of risks involved in (naïve) supply chain management.

4.2.1 Scenario 1: Single shock added on a constant demand rate

Warburton (2004) was the first to prove that a shock of the final demand causes a higher production rate of the manufacturer than the demand of the preceding levels of the chain, and the Bullwhip effect declines over time. In this study we focus on the cost function being of manager's great interest due their goal of minimizing overall cost. Note that the Bullwhip effect is influenced by the selected predictor, too. The total cost of operating such a supply chain applying a MMSE predictor amount to $TCSC_{MMSE}$ =6.300 money units. Note, that the total cost sum up to $TSSC_{exsm}$= 6.645 if an exponential smoothing, and to $TSSC_{movavg}$= 7.196 if a moving average predictor is used.

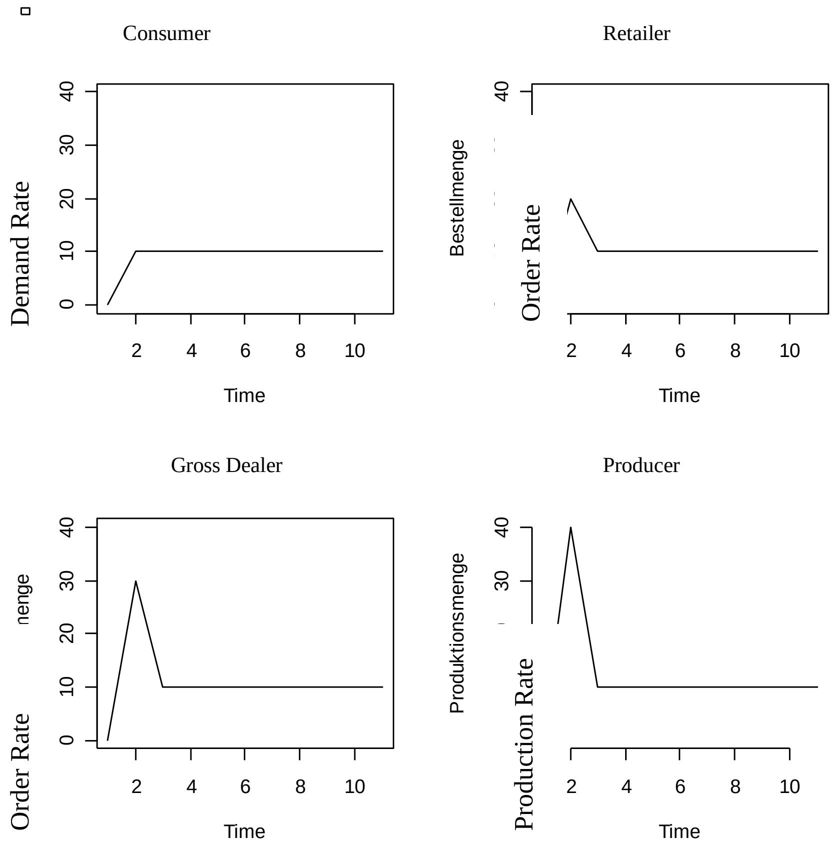

Figure 3. Effects of a constant demand rate with a superimposed shock on the order rates and production rate

4.2.2 Scenario 2: Stationary non-seasonal demand rate

Here we model the demand process as an AR_1MA_1 with AR and MA parameter $\alpha = 0{,}7$ and $\beta = 0{,}3$. The generated time-series of the demand rate under such a regime and the corresponding order and production rates are represented in Figure 4. As can be seen below, the final demand rates range between 30 and 35 units, while the deviations of the order rates of the succeeding chain levels increase causing a range of the production rates from 25 to 41 units. The Bullwhip effect is $Var(X_t)/Var(D_t)$ = 8,4. This means that the rate of production of a producer is about eight times higher than the related rate of the end consumer. Furthermore, the inventory of the producer is four times bigger than the end user's demand rate. Both indicators signal a large risk to the management.
The total cost of the supply chain amounts to TCSC= 20.655.

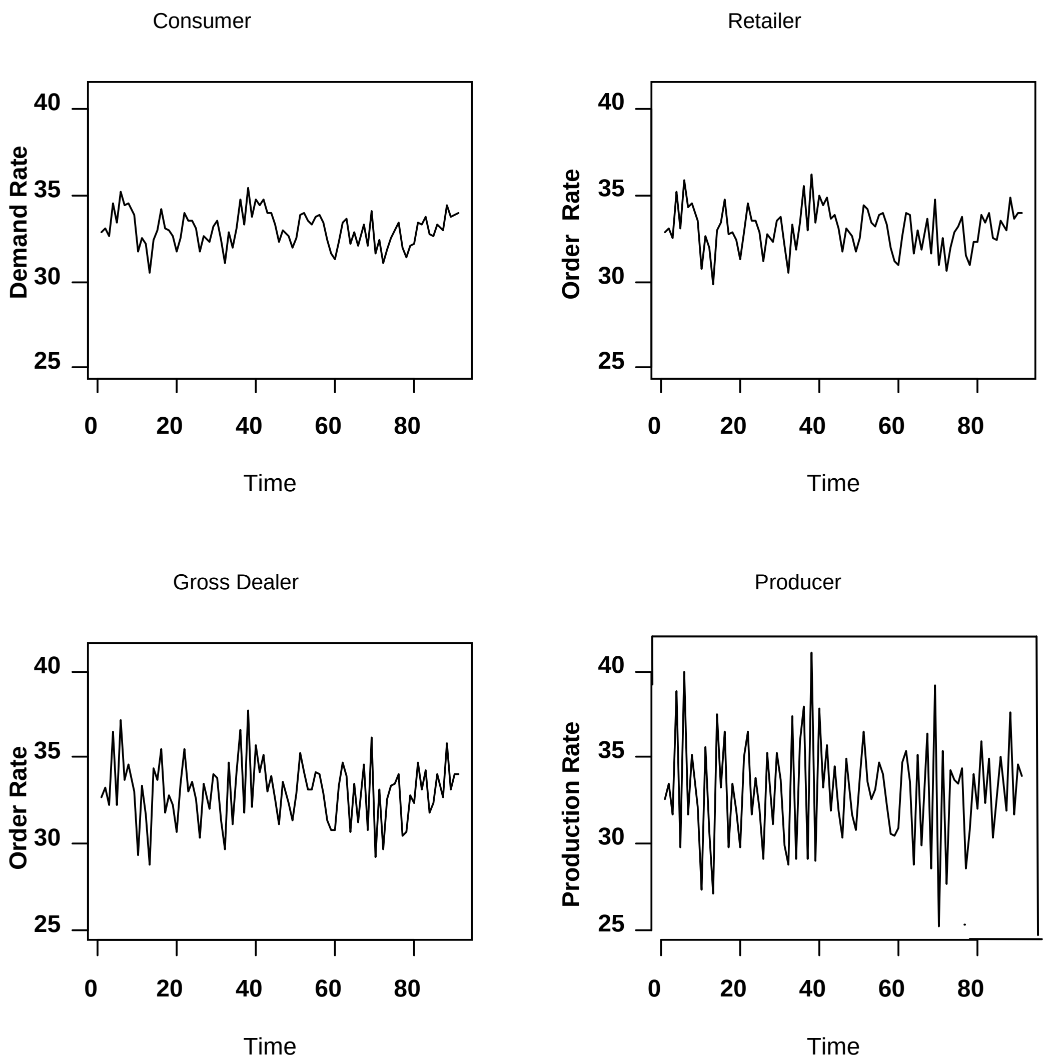

Figure 4. Effects of stationary, non-seasonal AR_1MA_1 demand rate on the order rates and production rate ($\alpha = 0{,}7$ and $\beta = 0{,}3$)

4.2.3 Scenario 3: Stationary non-seasonal demand rate

Again we model the demand process as an AR_1MA_1 with $\alpha = 0{,}7$ and $\beta = 0{,}3$. We are interested in the lead time or production time. As can be seen from Figure 5 the Bullwhip effect increases more or less linear with an increase of the delivery (lead) time of the gross dealer. This effect is due to the fact that the producer is the last member of the supply chain, and his production runs without interrupts or capacity restrictions.
From Figure 6 is becomes evident that an increase of the Bullwhip effect leads to about linear-increasing cost.

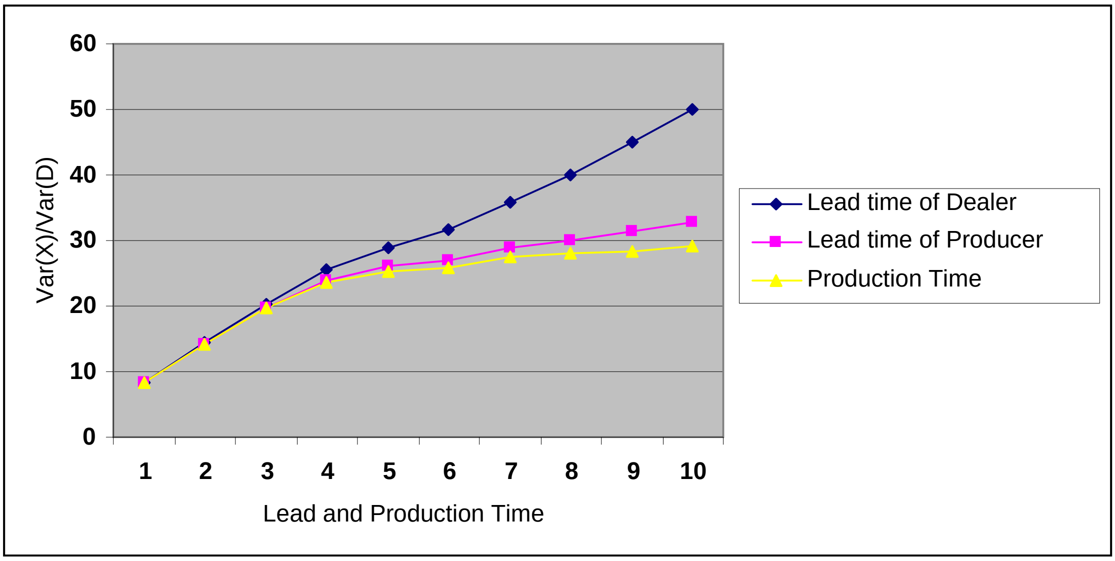

Figure 5. The Bullwhip Effect dependent upon lead and production time ($\alpha = 0{,}7$ and $\beta = 0{,}3$)

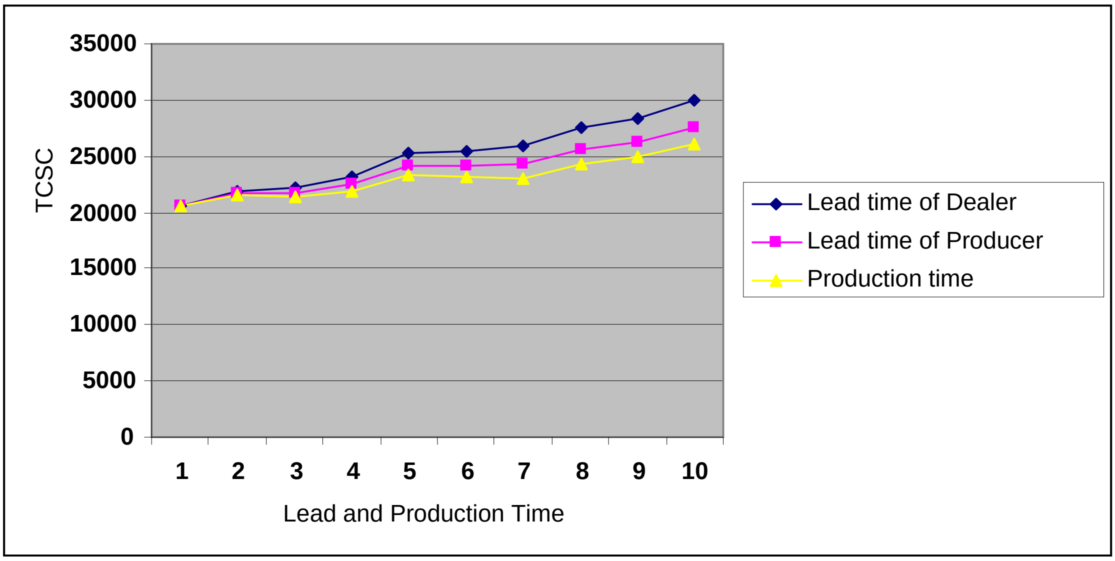

Figure 6. Total Cost of supply chain dependent upon lead and production time ($\alpha = 0{,}7$ and $\beta = 0{,}3$)

4.2.4 Scenario 4: Stationary seasonal demand rate

It is intuitive that the cyclic behaviour of a supply change should increase if the underlying final demand rate D_t shows a seasonal pattern. We can simply emulate such a seasonal time-series by a specially parameterized autoregressive moving-average process, i.e. AR_4MA_2.

(23) $D_t = \delta + \alpha D_{t-4} + u_t - \beta u_{t-2}$.

The setting $\alpha = 0{,}6$ and $\beta = -0{,}5$ leads to a seasonal pattern. As becomes clear from Figure 7 the Bullwhip effect caused by such a (for instance quarterly) demand rate is $Var(X_t)/Var(D_t)_{saisonal}$ = 77,22. Note, that the related non-seasonal effect is $Var(X_t)/Var(D_t)_{non\text{-}seasonal}$ = 19,96. Thie next relationship corresponds to cost involved. The total cost for a seasonal process is $TCSC_{seasonal}$ = 18.295, a non-seasonal demand process reduces to $TCS_{Cnon\text{-}seasonal}$ = 16.531 < $TCSC_{seasonal}$.

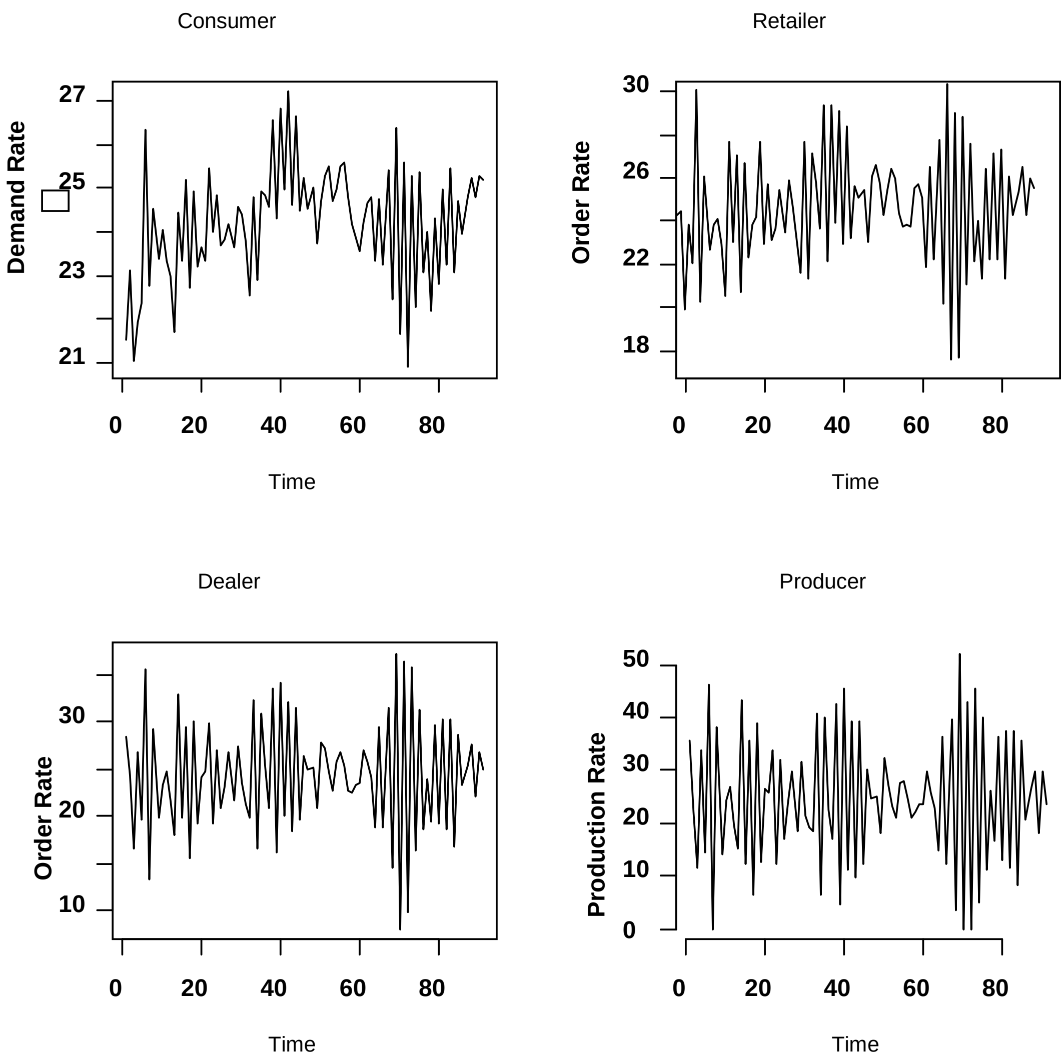

Figure 7. Effects of a special stationary, seasonal AR_4MA2_1 demand rate on order rates and production rate ($\alpha = 0,6$ and $\beta = -0,5$)

4.2.5 Scenario 5: Non-stationary, cyclic demand rate

Now we specify the demand rate D_t as a "curtailed" AR_2 process with $\alpha_1 = -0,75$ and $\alpha_2 = 0,5$ in order to avoid negative values of the demand rate. This parameter selection gives further insight in the characteristics of a supply chain in so far as it leads to cyclic, non-stationary behaviour, cf. Figure 8. Let:

(24) $$D_t = \delta - 0,75 D_{t-1} + 0,5 D_{t-2} + u_t \text{ if } D_t \geq 0, \text{ else zero.}$$

We assume that the supply chain members use a predictor with minimum quadratic errors (MMSE). Consequently, the retailer applies the one-step ahead predictor for D_{t+1} in t:

(25) $$\hat{D}_t(1) = \delta + \alpha_1 D_t + \alpha_2 D_{t-1}.$$

The succeeding levels of the chain are only affected by the underlying demand process. The gross dealer has to predict the retailer's demand rate O_t^E as follows:

(26) $$\hat{O}_t^E(1) = \delta + \alpha_1 O_t^E + \alpha_2 O_{t-1}^E$$

and the producer's predictor for the gross dealer's order rate O_t^G is:

(27) $$\hat{O}_t^G(1) = \delta + \alpha_1 O_t^G + \alpha_2 O_{t-1}^G .$$

The order rate, the inventory and the work-in-process are not affected.

It is well known that an AR_2-process is of stationary type iff the inertia conditions are stationary and the following restrictions on the parameter space are fulfilled

(28) $1-\alpha_1-\alpha_2>0$, $1+\alpha_1-\alpha_2>0$, and $1+\alpha_2>0$.

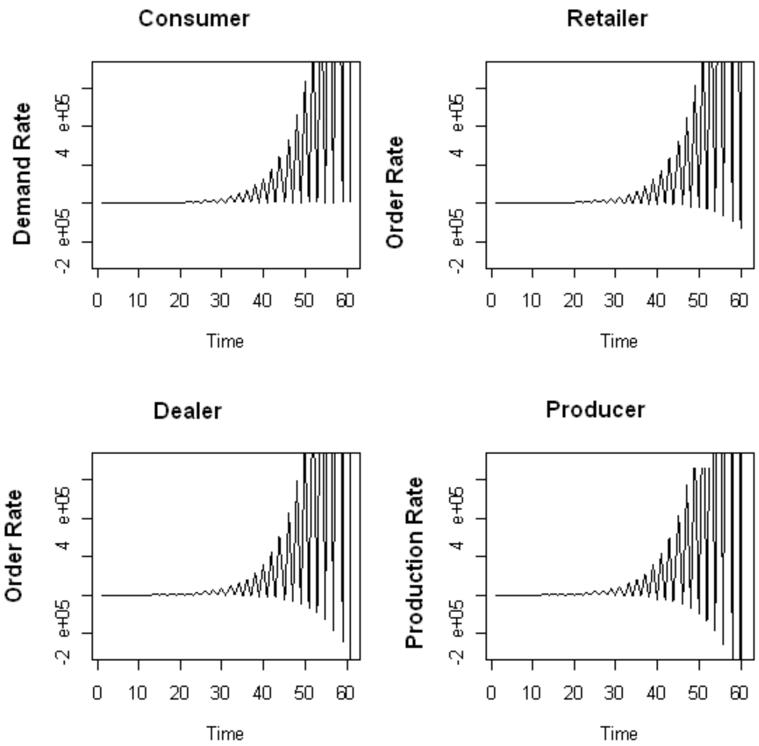

Figure 7. Effects of a non-stationary, "curtailed" AR_2 demand rate on the order rates and production rate ($\alpha_1 = -0{,}75$ and $\alpha_2 = 0{,}5$)

We get the simulated bullwhip effect $Var(X_t)/Var(D_{t\ non\text{-}stationary}) = 3$. The total cost of the supply chain increase to a level of TCSC = 151.235.

5 CONCLUSION

Our résumé is that vendors as well supply chain managers of SCM software should have an eye on the risks involved in operating such chains and should carefully select the many control parameters like predictor, order or production time and inventory strategy of a supply chain during implementation and management.

From Scenario 1 one can learn the lesson that a “good” predictor pays out even for constant demand rates superimposed by a single shock. Scenario 2 shows clearly, that even in case of a non-seasonal demand rate the Bullwhip effect can be amplified by a factor 8! Scenario 3 gives evidence that lead and production times behave approximately linear under a stationary, non-seasonal demand rate regime. Of course, we learn from scenario 4 that seasonality makes everything more badly. Finally, a non-stationary demand rate leads to a blow-up of cost.

Summarizing, a right-customizing of SCM software is necessary during its early operating phase, and later an effective monitoring of the Bullwhip and the related main business indicators is mandatory.

References

Chen, F.; Drezner, Z.; Ryan, J. K.; Simchi-Levi, D. 2000. ’Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information’ Management Science, 46(3): 436-443.

Disney, S. M.; Towill, D. R. 2003.’On the bullwhip and inventory variance produced by an ordering policy’. International Journal of Management Science 31: 157-67.

Duc, T. T. H.; Luong, H. T.; Kim, Y. 2008. ‘A measure of bullwhip effect in supply chains with a mixed autoregressive-moving average demand process’. European Journal of Operational Research 187: 243-256.

Kaihla, P. 2002. ‘Inside Cisco's $2 Billion Blunder’. Business 2.0.

Keller, S. 2004. ‚Die Reduzierung des Bullwhip-Effektes’. PhD diss. Universität Duisburg-Essen, Deutscher Universitäts-Verlag.

Lee, H. L.; Padmanabhan, V.; Whang, S. 1997. ‘The Bullwhip Effect in Supply Chains’. Sloan Management Review, 93-102.

Lenz, H. - J. and Essebbabi, D. 2005. ‘Analytische und numerische Untersuchungen zum Peitscheneffekt im Supply Chain Management’. Discussion Paper No 2005/24, Fachbereich Wirtschaftswissenschaft, Freie Universität Berlin, ISBN 3-938369-23-X

Paustian, E. 2009. ‘MC Simulation of the Bullwhip Effect for Supply Chain Management’. MS thesis, Department of Production, Information Systems and Operations Research, Freie Universität Berlin

Warburton, D. 2004. ‘An Analytical Investigation of the Bullwhip Effect’. Production and Operations Management Society. 13(2): 150-160.